HP COMPUTER CURRICULUM

Geometrical Optics

TEACHERS ADVISOR

HEWLETT **hp** PACKARD

Hewlett-Packard
Computer Curriculum Series

physics
TEACHER'S ADVISOR

geometrical
optics

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This material is designed to be used with any Hewlett-Packard system with the BASIC programming language such as the 9830A, Educational BASIC, and the 2000 and 3000 series systems.

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INTRODUCTION

This Physics Set of the Hewlett-Packard Computer Curriculum Series consists of a set of a Student Lab Book and a corresponding Teacher's Advisor. It was designed to help meet the need for computer-oriented problems in physics, providing students an opportunity to use a computer as a problem solving tool within a particular subject matter area.

The materials are designed for flexible use as desired by the individual instructor. The material and exercises in this unit are intended as an "enrichment" experience in the field of geometrical optics. Fermat's principle is used to discover the laws of reflection and refraction. Normally this requires differential calculus. However, with the speed of the computer, the same task requires only algebra and some trigonometry and therefore can be used at the introductory college level and in high school. The final topic covered is the method of ray tracing as applied to thin lenses. Since neither of the topics above is usually covered in the introductory texts, the use of this material will not compete with your text. Instead, it can be used to supplement and enrich in any fashion you choose.

The degree of difficulty of the material is dependent upon the amount of assistance which you choose to provide. With no assistance, the better physics student should be challenged. However, given a good deal of assistance, any physics student should be able to work out the exercises with no great difficulty. The level of the material is determined by the assumption that students taking introductory physics will be quite capable as a group.

The Lab Book provides text material and programming exercises for the students, a sample program and advanced problems. The Teacher's Advisor contains an example of a program to solve each exercise and a brief discussion of the important elements of the exercise.

For best results, you should study all the solutions until you are certain you have a complete grasp of the general methods. This should be done before assigning the material to the class. Generally, the exercises are cumulative so that as techniques are developed they are used in subsequent exercises. Therefore, you will probably wish to proceed through the exercises in the order in which they are given.

You will undoubtedly think of different programming methods or techniques as you study the example programs. Encourage the students to do the same. There are no *approved* solutions. All solutions are acceptable if they produce the correct results. At this level, there is no need for emphasis on the efficiency of a student's program. The important question is, does it work?

NOTES

REVIEW OF COORDINATE GEOMETRY

Exercise 1 — Coordinates

This is designed solely to provide practice in the location of points on a two-dimensional cartesian coordinate system. If your students do not need this practice, go on to Exercise 2. The solution follows.

Point	x	y
A	1	1
B	0	−2
C	−4	0
D	0	0
E	5.5	0.5
F	3	4.5
G	−2	3
H	−4.5	−3
I	2.5	−4
J	0	5

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Exercise 2 — Distance Between Points

The only question to be decided in this exercise is how to control the looping through the data. The program below uses a FOR loop. Other methods are possible. In line 110, N is the number of line segments to be processed. Input of each segment takes place in line 130 with the computation in line 140.

```
100   REM DISTANCE BETWEEN TWO POINTS
110   READ N
120   FOR I=1 TO N
130   READ X1,Y1,X2,Y2
140   LET D=SQR((X2-X1)†2+(Y2-Y1)†2)
150   PRINT D
160   NEXT I
800   DATA 10
801   DATA 1,1,5,6
802   DATA -5,2,1,-2
803   DATA -3.5,-1.2,2.8,4.1
804   DATA 0,0,0,1.4283
805   DATA 0,-6.732,0,0
806   DATA -2,-3,-3,-2
807   DATA .923,-.149,-.758,-.358
808   DATA 0,1.414,1.414,0
809   DATA 2.8,1.5,-4.1,-2.1
810   DATA 0,3.6,0,-2.4
999   END
```

RUN

```
6.40312
7.2111
8.23286
1.4283
6.732
1.41421
1.69394
1.9997
7.78267
6
```

FERMAT'S PRINCIPLE

REFLECTION

Exercise 3 – Rays Between Fixed Points

The index of refraction is set equal to 1 in line 90. This is not required in this particular problem, but will be needed in subsequent ones. Hence, we establish the pattern here. In lines 100 through 130 the optical path P∅ is computed for the first integer value of X, the value of X = 1 being assigned at the same time to the variable X∅ which will be developed as the reflection point. A new optical path is computed in lines 150 through 170 each time through the loop. In lines 180 through 190 the new optical path is compared to the smallest of the previous optical paths. If the new one is smaller, we update P∅ and X∅.

```
90    LET N=1
100    LET D1=SQR((1-1)↑2+(0-4)↑2)
110    LET D2=SQR((11-1)↑2+(6-0)↑2)
120    LET P0=N*D1+N*D2
130    LET X0=1
140    FOR X=2 TO 11
150    LET D1=SQR((X-1)↑2+(0-4)↑2)
160    LET D2=SQR((11-X)↑2+(6-0)↑2)
170    LET P1=N*D1+N*D2
180    IF P1>P0 THEN 210
190    LET P0=P1
200    LET X0=X
210    NEXT X
220    PRINT "REFLECTION AT X = "X0
999    END
```

RUN

REFLECTION AT X =   5

Exercise 4 – Rays Between Changing Points

This is a simple modification of the program in Exercise 3. The initial and final points are read into the program in line 120. The appropriate changes are then carried through the balance of the program.

```
100    LET N=1
120    INPUT X1,Y1,X2,Y2
130    LET D1=Y1
140    LET D2=SQR((X2-X1)↑2+Y2↑2)
150    LET P0=N*D1+N*D2
160    LET X0=X1
170    FOR X=X1+1 TO X2
180    LET D1=SQR((X-X1)↑2+Y1↑2)
190    LET D2=SQR((X2-X)↑2+Y2↑2)
200    LET P1=N*D1+N*D2
210    IF P1>P0 THEN 240
220    LET P0=P1
230    LET X0=X
240    NEXT X
250    PRINT "REFLECTION AT X = "X0
999    END
```

RUN

```
?0,20,10,20
REFLECTION AT X =    5
```

Exercise 5 – Precision Computation

S (in line 100) is the initial step size in the search. In line 110, M (number of significant digits in answer desired) is set. The search procedure in lines 120 through 280 is the same as in Exercise 4. The only difference is found in line 180 which sets a counter C used to test against M. The value of the counter is tested in line 290. If less than M the search limits are reset in lines 300 through 330. Then the program loops back to the search procedure.

```
100    LET  S=1
110    INPUT  M
120    LET  N=1
130    INPUT  X1,Y1,X2,Y2
140    LET  D1=Y1
150    LET  D2=SQR((X2-X1)↑2+Y2↑2)
160    LET  P0=N*D1+N*D2
170    LET  X0=X1
180    LET  C=1
190    LET  A=X1+1
200    LET  B=X2
210    FOR  X=A  TO  B  STEP  S
220    LET  D1=SQR((X-X1)↑2+Y1↑2)
230    LET  D2=SQR((X2-X)↑2+Y2↑2)
240    LET  P1=N*D1+N*D2
250    IF  P1>P0  THEN  280
260    LET  P0=P1
270    LET  X0=X
280    NEXT  X
290    IF  C  >=  M  THEN  350
300    LET  A=X0-S
310    LET  B=X0+S
320    LET  S=S/10
330    LET  C=C+1
340    GOTO  210
350    PRINT  "REFLECTION AT X = "X0
999    END
```

RUN

?4
?0,10,10,8
REFLECTION AT X =   5.56101

RUN

?4
?-10,10,10,10
REFLECTION AT X = -1.81608E-08

RUN

?4
?5,10,10,15
REFLECTION AT X =   7.00401

Exercise 6 — Angles of Incidence and Reflection

The procedure in lines 100 through 340 is the same as in Exercise 5. Angles of incidence and reflection are computed in lines 350 and 360. These angles are converted to degrees and output in lines 370 and 380.

```
100   LET  S=1
110   INPUT  M
120   LET  N=1
130   INPUT  X1,Y1,X2,Y2
140   LET  D1=Y1
150   LET  D2=SQR((X2-X1)†2+Y2†2)
160   LET  P0=N*D1+N*D2
170   LET  X0=X1
180   LET  C=1
190   LET  A=X1+1
200   LET  B=X2
210   FOR  X=A  TO  B  STEP  S
220   LET  D1=SQR((X-X1)†2+Y1†2)
230   LET  D2=SQR((X2-X)†2+Y2†2)
240   LET  P1=N*D1+N*D2
250   IF  P1>P0  THEN  280
260   LET  P0=P1
270   LET  X0=X
280   NEXT  X
290   IF  C  >=  M  THEN  350
300   LET  A=X0-S
310   LET  B=X0+S
320   LET  S=S/10
330   LET  C=C+1
340   GOTO  210
350   LET  I=ATN((X0-X1)/Y1)
360   LET  R=ATN((X2-X0)/Y2)
370   PRINT  "ANGLE  OF  INCIDENCE  =  "57.2957*I"DEGREES"
380   PRINT  "ANGLE  OF  REFLECTION  ="57.2957*R"DEGREES"
999   END
```

```
RUN


?4
?0,10,20,5
ANGLE  OF  INCIDENCE  =   53.1335      DEGREES
ANGLE  OF  REFLECTION  =  53.1232      DEGREES
```

Exercise 7 — Discovery

The program in Exercise 6 is modified here to process several problems. The number of problems Z is input in line 95. The problem loop is established in lines 96 and 400. Inside this loop, the details are the same as in Exercise 6.

```
95    READ Z
96    FOR J=1 TO Z
100   LET S=1
110   LET M=3
120   LET N=1
130   READ X1,Y1,X2,Y2
140   LET D1=Y1
150   LET D2=SQR((X2-X1)↑2+Y2↑2)
160   LET P0=N*D1+N*D2
170   LET X0=X1
180   LET C=1
190   LET A=X1+1
200   LET B=X2
210   FOR X=A TO B STEP S
220   LET D1=SQR((X1-X)↑2+Y1↑2)
230   LET D2=SQR((X2-X)↑2+Y2↑2)
240   LET P1=N*D1+N*D2
250   IF P1>P0 THEN 280
260   LET P0=P1
270   LET X0=X
280   NEXT X
290   IF C >= M THEN 350
300   LET A=X0-S
310   LET B=X0+S
320   LET S=S/10
330   LET C=C+1
340   GOTO 210
350   LET I=ATN((X0-X1)/Y1)
360   LET R=ATN((X2-X0)/Y2)
370   LET I=57.2957*I
380   LET R=57.2957*R
390   PRINT X1;Y1;X2;Y2;I,R
400   NEXT J
800   DATA 5
801   DATA -10,5,8,4
802   DATA 2,10,15,10
803   DATA 0,5,10,10
804   DATA 20,25,30,15
805   DATA -20,6,-10,10
999   END
```

RUN

-10	5	8	4	63.4577	63.4062
2	10	15	10	33.0238	33.0238
0	5	10	10	33.6636	33.7032
20	25	30	15	14.0362	14.0363
-20	6	-10	10	32.0051	32.0055

Exercise 8 — Generalization

In this exercise, the student is asked to "discover" the law of specular reflection. Note carefully that nothing in our development mentions the fact that the incident and reflected rays lie in the same plane. This certainly has been implied since we have been working in a coordinate plane. As an optional exercise, students might be asked to repeat the development above in a three-dimensional coordinate system in which possible paths outside of a single plane could be investigated. Of course the student will discover that the rays do lie in a single plane.

Exercise 9 — Corner Reflection

This should be considered an optional exercise for the better student. Since two reflections are involved, a double loop is required to conduct the search. Trace the program carefully to see how the search is carried out. The object is to let the student discover the characteristics of a corner reflector. Have the students plot their results and discover that after two reflections, the ray is returning parallel to the initial ray.

```
100    LET  S=1
110    INPUT  M
120    LET  N=1
130    INPUT  X1,Y1,X2,Y2
140    LET  D1=X1
150    LET  D2=Y1
160    LET  D3=SQR(X2↑2+Y2↑2)
170    LET  P0=N*(D1+D2+D3)
180    LET  X0=0
190    LET  Y0=Y1
200    LET  C=1
```

```
210     LET A1=Y1-1
220     LET B1=0
230     LET A2=0
240     LET B2=X2
250     FOR Y=A1 TO B1 STEP -S
260     FOR X=A2 TO B2 STEP S
270     LET D1=SQR((Y1-Y)↑2+X1↑2)
280     LET D2=SQR(Y↑2+X↑2)
290     LET D3=SQR(Y2↑2+(X2-X)↑2)
300     LET P1=N*(D1+D2+D3)
310     IF P1>P0 THEN 350
320     LET P0=P1
330     LET X0=X
340     LET Y0=Y
350     NEXT X
360     NEXT Y
370     IF C >= M THEN 450
380     LET A1=Y0+S
390     LET B1=Y0-S
400     LET A2=X0-S
410     LET B2=Y0+S
420     LET S=S/10
430     LET C=C+1
440     GOTO 250
450     PRINT "REFLECTION AT"
460     PRINT 0,Y0;"AND"X0,0
999     END
```

```
RUN


?3
?5,10,10,5
REFLECTION AT
   0                5.            AND 5.                0




RUN


?3
?2,5,4,3
REFLECTION AT
   0                2.33          AND 1.75              0
```

REFRACTION

Exercise 10 — Refraction With Precision Computation

The modifications from the program in Exercise 5 follow from the discussion in the student's Lab Book.

```
100    LET S=1
110    INPUT M
120    INPUT N1,N2
130    INPUT X1,Y1,X2,Y2
140    LET D1=Y1
150    LET D2=SQR((X2-X1)↑2+Y2↑2)
160    LET PO=N1*D1+N2*D2
170    LET XO=X1
180    LET C=1
190    LET A=X1+1
200    LET B=X2
210    FOR X=A TO B STEP S
220    LET D1=SQR((X-X1)↑2+Y1↑2)
230    LET D2=SQR((X2-X)↑2+Y2↑2)
240    LET P1=N1*D1+N2*D2
250    IF P1>PO THEN 280
260    LET PO=P1
270    LET XO=X
280    NEXT X
290    IF C >= M THEN 350
300    LET A=XO-S
310    LET B=XO+S
320    LET S=S/10
330    LET C=C+1
340    GOTO 210
350    PRINT "REFRACTION AT X = "XO
999    END
```

```
RUN

?4
?1,1.33
?0,10,20,-10
REFRACTION AT X =   12.693


RUN

?4
?1.33,1
?0,10,20,-10
REFRACTION AT X =   7.32201


RUN

?4
?1,1
?0,10,20,-10
REFRACTION AT X =   10
```

Exercise 11 – Refraction Between Different Media

Following are runs of the program shown for Exercise 10 using the values given in Exercise 11.

```
RUN

?4
?1.48,1.33
?-10,10,10,-10
REFRACTION AT X = -1.048


RUN

?4
?1.33,1.65
?-10,10,10,-10
REFRACTION AT X =   2.085
```

```
RUN
?4
?1.00,1.65
?-10,10,10,-10
REFRACTION AT X =   4.28901
```

```
RUN
?4
?1.65,1.00
?-10,10,10,-10
REFRACTION AT X = -4.27399
```

```
RUN
?4
?1,1
?-10,10,10,-10
REFRACTION AT X = -1.81608E-08
```

Exercise 12 — Angles of Incidence and Refraction

The changes in the program used for Exercise 10 are specified in the student's Lab Book.

```
100    LET  S=1
110    INPUT  M
120    INPUT  N1,N2
130    INPUT  X1,Y1,X2,Y2
140    LET  D1=Y1
150    LET  D2=SQR((X2-X1)↑2+Y2↑2)
160    LET  P0=N1*D1+N2*D2
170    LET  X0=X1
180    LET  C=1
190    LET  A=X1+1
200    LET  B=X2
210    FOR  X=A  TO  B  STEP  S
220    LET  D1=SQR((X-X1)↑2+Y1↑2)
230    LET  D2=SQR((X2-X)↑2+Y2↑2)
240    LET  P1=N1*D1+N2*D2
250    IF  P1>P0  THEN  280
260    LET  P0=P1
270    LET  X0=X
280    NEXT  X
290    IF  C  >=  M  THEN  350
300    LET  A=X0-S
310    LET  B=X0+S
320    LET  S=S/10
330    LET  C=C+1
340    GOTO  210
350    PRINT  "REFRACTION  AT  X  =  "X0
360    LET  I=57.2957*ATN((X0-X1)/Y1)
370    LET  R=57.2957*ATN((X2-X0)/Y2)
380    PRINT  "ANGLE  OF  INCIDENCE  =  "ABS(I)"DEGREES"
390    PRINT  "ANGLE  OF  REFRACTION  =  "ABS(R)"DEGREES"
999    END
```

```
RUN
?4
?1.48,1.33
?-10,10,10,-10
REFRACTION AT X = -1.048
ANGLE OF INCIDENCE =  41.8348    DEGREES
ANGLE OF REFRACTION =  47.8504    DEGREES


RUN
?4
?1.33,1.65
?-10,10,10,-10
REFRACTION AT X =  2.085
ANGLE OF INCIDENCE =  50.3931    DEGREES
ANGLE OF REFRACTION =  38.3616    DEGREES


RUN
?4
?1.00,1.65
?-10,10,10,-10
REFRACTION AT X =  4.28901
ANGLE OF INCIDENCE =  55.0141    DEGREES
ANGLE OF REFRACTION =  29.7306    DEGREES


RUN
?4
?1.65,1.00
?-10,10,10,-10
REFRACTION AT X = -4.27399
ANGLE OF INCIDENCE =  29.7955    DEGREES
ANGLE OF REFRACTION =  54.9858    DEGREES


RUN
?4
?1,1
?-10,10,10,-10
REFRACTION AT X = -1.81608E-08
ANGLE OF INCIDENCE =  44.9999    DEGREES
ANGLE OF REFRACTION =  44.9999    DEGREES
```

Exercise 13 — Discovery

The program below is a simple modification of the program used for Exercise 12.
The modifications are specified in the Lab Book.

```
100    LET S=1
110    INPUT M
120    INPUT N1,N2
130    INPUT X1,Y1,X2,Y2
140    LET D1=Y1
150    LET D2=SQR((X2-X1)↑2+Y2↑2)
160    LET P0=N1*D1+N2*D2
170    LET X0=X1
180    LET C=1
190    LET A=X1+1
200    LET B=X2
210    FOR X=A TO B STEP S
220    LET D1=SQR((X-X1)↑2+Y1↑2)
230    LET D2=SQR((X2-X)↑2+Y2↑2)
240    LET P1=N1*D1+N2*D2
250    IF P1>P0 THEN 280
260    LET P0=P1
270    LET X0=X
280    NEXT X
290    IF C >= M THEN 350
300    LET A=X0-S
310    LET B=X0+S
320    LET S=S/10
330    LET C=C+1
340    GOTO 210
350    PRINT "REFRACTION AT X = "X0
360    LET I=ABS(ATN((X0-X1)/Y1))
370    LET R=ABS(ATN((X2-X0)/Y2))
380    PRINT "ANGLE OF INCIDENCE = "57.2957*I"DEGREES"
390    PRINT "ANGLE OF REFRACTION= "57.2957*R"DEGREES"
400    PRINT N1*I,N1*TAN(I),N1*COS(I),N1*SIN(I)
410    PRINT N2*R,N2*TAN(R),N2*COS(R),N2*SIN(R)
999    END
```

```
RUN
?3
?1.48,1.33
?-10,10,10,-10
REFRACTION AT X = -1.05
ANGLE OF INCIDENCE =   41.8285     DEGREES
ANGLE OF REFRACTION=   47.8555     DEGREES
   1.08047          1.3246        1.10281          .987018
   1.11087          1.46965       .892431          .986137


RUN
?3
?1.33,1.65
?-10,10,10,-10
REFRACTION AT X =   2.08
ANGLE OF INCIDENCE =   50.3815     DEGREES
ANGLE OF REFRACTION=   38.3792     DEGREES
   1.1695           1.60664       .848104          1.02451
   1.10524          1.3068        1.29347          1.02443


RUN
?3
?1.00,1.65
?-10,10,10,-10
REFRACTION AT X =   4.28
ANGLE OF INCIDENCE =   54.9971     DEGREES
ANGLE OF REFRACTION=   29.7695     DEGREES
   .959882          1.428         .573616          .819124
   .857302          .9438         1.43225          .819246


RUN
?3
?1.65,1.00
?-10,10,10,-10
REFRACTION AT X = -4.27999
ANGLE OF INCIDENCE =   29.7695     DEGREES
ANGLE OF REFRACTION=   54.9971     DEGREES
   .857302          .943801       1.43225          .819247
   .959882          1.428         .573617          .819124
```

Exercise 14 — Generalization

The student should come up with some version of Snell's Law at this point.

Exercise 15 — Double Refraction

This program contains essentially the same search program as used in Exercise 10.
The significant difference is that a double loop is required.

```
100    LET S=1
110    INPUT M
120    INPUT N1,N2,N3
130    INPUT X1,Y1,X2,Y2
140    INPUT C1,C2
150    LET D1=Y1-C1
160    LET D2=C1-C2
170    LET D3=SQR((X2-X1)↑2+(Y2-C2)↑2)
180    LET P0=N1*D1+N2*D2+N3*D3
190    LET X8=X1
200    LET X9=X1
210    LET C=1
220    LET A1=X1+1
230    LET B1=X2
240    LET A2=X1
250    LET B2=X2
260    FOR U=A1 TO B1 STEP S
270    FOR V=A2 TO B2 STEP S
280    LET D1=SQR((U-X1)↑2+(Y1-C1)↑2)
290    LET D2=SQR((V-U)↑2+(C1-C2)↑2)
300    LET D3=SQR((X2-V)↑2+(Y1-C1)↑2)
310    LET P1=N1*D1+N2*D2+N3*D3
320    IF P1>P0 THEN 360
330    LET P0=P1
340    LET X8=U
350    LET X9=V
360    NEXT V
370    NEXT U
380    IF C >= M THEN 460
390    LET A1=X8-S
400    LET B1=X8+S
410    LET A2=X9-S
420    LET B2=X9+S
430    LET S=S/10
440    LET C=C+1
450    GOTO 260
460    PRINT "RAY PASSES THROUGH"
470    PRINT X8;C1"AND      "X9;C2
999    END
```

```
RUN
?3
?1,1.33,1
?-10,10,10,-10
?2,-2
RAY PASSES THROUGH
-1.32        2      AND      1.33        -2
```

Exercise 16 — A Complex Problem

This program, like the one in Exercise 15, requires a double loop in the search. Essentially it is the same program, with a few geometrical differences.

```
100    LET S=1
110    INPUT M
120    INPUT N1,N2
130    INPUT X1,Y1,X2,Y2
140    LET D1=Y1
150    LET D2=-X1
160    LET D3=SQR(X2↑2+Y2↑2)
170    LET P0=N1*(D1+D2)+N2*D3
180    LET X0=X1
190    LET Y0=0
200    LET C=1
210    LET A1=X1+1
220    LET B1=0
230    LET A2=0
240    LET B2=Y2
250    FOR X=A1 TO B1 STEP S
260    FOR Y=A2 TO B2 STEP S
270    LET D1=SQR((X-X1)↑2+Y1↑2)
280    LET D2=SQR(X↑2+Y↑2)
290    LET D3=SQR(X2↑2+(Y2-Y)↑2)
300    LET P1=N1*(D1+D2)+N2*D3
310    IF P1>P0 THEN 350
320    LET P0=P1
330    LET X0=X
340    LET Y0=Y
350    NEXT Y
360    NEXT X
370    IF C >= M THEN 450
380    LET A1=X0-S
390    LET B1=X0+S
400    LET A2=Y0-S
```

```
410    LET  B2=Y0+S
420    LET  S=S/10
430    LET  C=C+1
440    GOTO 250
450    PRINT "RAY PASSES THROUGH "
460    PRINT X0,0"AND      "0;Y0
999    END


RUN
?3
?1,1.5
?-10,10,10,10
RAY PASSES THROUGH
-2.66              0     AND        0      3.63
```

NOTES

RAY TRACING

Exercise 17 – A Single Lens

For the space:

$$\theta_2 = \theta_1 = -0.1$$

$$y_2 = y_1 + s\theta_1 = 1\ 3\ (20)(-0.2) = -3$$

This ray is then carried through the lens:

$$\theta_2 = \theta_1 - y_1/f = (-0.1) - (-3)/10 = +0.2$$

$$y_2 = y_1 = -3$$

The output ray has $\theta = 0.2$ and $y = -3$.

Exercise 18 – Two Lenses

The results of Exercise 17 are input into the space equations:

$$\theta_2 = \theta_1 = 0.2$$

$$y_2 = y_1 + s\theta_1 = (-3) + (10)(0.2) = -1$$

This ray is input into the lens equations to get the final result.

$$\theta_2 = \theta_1 - y_1/f = (0.2) - (-1)/(-20) = -0.15$$

$$y_2 = y_1 = -1$$

The output ray has $\theta = 0.15$ and $y = -1$.

Exercise 19 — A Program for N Lenses

Line 100 contains the description of the initial ray which enters the optical system. T stands for "Theta". N1 is the number of optical elements (either a space or a lens) in the system and is used to control the limit on the FOR loop opened in line 120. Inside the loop, the data for each element is read in, tested to see which equation is appropriate, and the ray then updated. After all the elements of the system have been processed the description of the output ray is typed out.

```
100   READ Y,T
110   READ N1
120   FOR I=1 TO N1
130   READ N2,A
140   IF N2=2 THEN 170
150   LET Y=Y+A*T
160   GOTO 180
170   LET T=T-Y/A
180   NEXT I
200   PRINT Y,T
210   DATA 0,.1
220   DATA 4
230   DATA 1,10
240   DATA 2,20
250   DATA 1,20
260   DATA 2,10
999   END
```

RUN

2   -.15

Exercise 20 — Intersection of Rays

Using the program from Exercise 19 the following results are obtained.

```
210 DATA 1,0
RUN
 0                    -.05

210 DATA 1,-.1
RUN
-2                     .1
```

Examining the results we see that the two rays differ in position by 2 units, have a relative angle between each other of 0.15, and are converging as they move away from the last element in the optical system. Thus the rays will intersect 2/0.15 = 13.33 units to the right of the last element in the system.

Exercise 21 — Description of Image

The program from Exercise 19 is modified with new data and run with the results below:

```
220 DATA 2
230 DATA 1,20
240 DATA 2,10

210 DATA 1,0
RUN
 1                    -.1

210 DATA 1,-.1
RUN
-1                     0
```

The two output rays are two units apart, have a relative angle between them of 0.1 and are converging. Therefore the image is *real*. The rays intersect 2/.1 = 20 units to the right 1 unit below the optical axis. Therefore the image is *inverted*, and the same size as the object.

NOTES

OPTIONAL MATERIAL

Exercise 22 — A Matrix Program

The matrix X is used to hold the input ray. Each of the matrices corresponding to an optical element is read into the system and multiplied into M, the system matrix. Finally the output ray Y is the product of the system matrix M and the input ray X.

```
100    DIM A[2,2],M[2,2],B[2,2],X[2,1],Y[2,1]
110    MAT   READ X
120    READ N
130    MAT M=IDN
140    FOR I=1 TO N
150    MAT   READ A
160    MAT B=M*A
170    MAT M=B
180    NEXT I
190    MAT Y=M*X
200    MAT   PRINT Y
210    DATA .1,0
220    DATA 4
230    DATA 1,-.1,0,1
240    DATA 1,0,20,1
250    DATA 1,-.0503,0,1
260    DATA 1,0,10,1
999    END
```

RUN
-.15

2

Exercise 23 — Description of Image

This exercise is a very good test of understanding of geometry and all the material that has preceded. The rules are given below:

Let the system matrix be described by four numbers as indicated.

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

If a & c have the same signs, image is virtual.

If a & c have opposite signs, image is real.

If $d > 0$ & $b > 0$ then the image is erect.

If $d > 0$ & $b < 0$ & $|d| > |bc/a|$ then the image is erect.

If $d > 0$ & $b < 0$ & $|d| < |bc/a|$ then the image is inverted.

If $d < 0$ & $b < 0$ then the image is inverted.

If $d < 0$ & $b > 0$ & $|d| > |bc/a|$ then the image is inverted.

If $d < 0$ & $b > 0$ & $|d| < |bc/a|$ then the image is erect.

If $|d - bc/a| > 1$ then the image is magnified.

If $|d - bc/a| = 1$ then the image is the same size.

If $|d - bc/a| < 1$ then the image is reduced in size.